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TÍTULO DO TRABALHO:

APPLICATION IN RESERVOIR SIMULATION OF A FINITE VOLUME METHOD LIBRARY USING UNSTRUCTURED GRIDS

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Abstract

In the oil industry it is necessary to use forecasts developed through simulators based on approximations to make reservoir management choices. Reservoir management integrates approaches and techniques for well design, recovery process, reservoir characterization, monitoring, and the focus of this paper, simulation. They rely on describing physical phenomena; for this, considerations and approximations are used to specify a studied scenario. The fluid flow simulation is fundamental to obtain a trustable prediction for production, for example. Reservoirs generally do not present a regular domain, and their shapes are far from fully known; this provides unreliability on the physical domain representation. Thus, it was identified that the unstructured grids better describe the non-regular domain of a reservoir, seeking the possibility of mesh modeling according to the physical characteristics. The numerical fluid flow in porous media discretization equations is a challenging problem on unstructured grids, which a methodology is presented in this work. The main objective of this paper is to solve a monophasic fluid flow petroleum reservoir problem with injection and production wells for an isotropic and an orthotropic medium. The approach taken into consideration is a simplified single-phase flow through porous media using a finite volume method for an unstructured grid. The implementation in Python leverages the PyEFVLib library, in which it presents functions that simplify solving the numerical discretization of partial differential equations. The results achieved demonstrate the applicability of this methodology (Element based Finite Volume Method - EbFVM) in reservoir simulation even with anisotropic permeability, which is usually exceedingly difficult to consider in traditional numerical methods that use structured grids. Successful simulations exhibit the pressure through an unstructured grid and the influence of the two well types for an isotropic and an orthotropic medium. They have also shown the usability of PyEFVLib for this utilization enhancing the importance of python, that is a programming language with wide collections ready to be used in scientific computing.

Keywords: Fluid Flow Simulation, Unstructured Grids, Element Based Finite Volume Method - EbFVM, PyEFVLib.

Introduction

Reservoir management in the oil industry is a complex and multifaceted task that requires the integration of various approaches and techniques to ensure efficient and sustainable extraction of hydrocarbons (KALAYDJIAN, 2002). Key aspects of reservoir management include well design, recovery processes, reservoir characterization, monitoring, and simulation. Among these, simulation stands out as a critical tool, enabling engineers and scientists to develop forecasts and make informed decisions based on approximations of the reservoir's behavior. These forecasts are typically generated using simulators that describe physical phenomena within the reservoir, employing various considerations and approximations to model the studied scenarios accurately.

Fluid flow simulation, in particular, is essential for obtaining reliable production predictions. As submitted by Hurtado (2011), the irregular nature of reservoir domains and the incomplete knowledge of their shapes introduce significant challenges in accurately representing the physical domain. In addition, traditional structured grids often fall short in capturing the complex geometries of reservoirs. In contrast, unstructured grids offer a more flexible and accurate description of non-regular domains by allowing mesh modeling to conform to the reservoir's physical characteristics. Despite their advantages, the numerical discretization of fluid-flow equations in porous media on unstructured grids remains a challenging problem.

This paper presents a methodology to address the numerical discretization of monophasic fluid flow in petroleum reservoirs with injection and production wells, considering both isotropic and orthotropic media. The study employs a simplified single-phase flow model through porous media using the Finite Volume Method (FVM) on an unstructured grid. The implementation is carried out in Python, utilizing the PyEFVLib library, which provides functions that simplify the numerical discretization of partial differential equations.

The primary objective of this research is to demonstrate PyEFVLib and the easy applicability of the Element based Finite Volume Method (EbFVM) in reservoir simulation, even when dealing with anisotropic permeability, which is particularly challenging to account for using traditional numerical methods that rely on structured grids. The results achieved through this methodology highlight the effectiveness of the EbFVM in simulating pressure distribution within unstructured grids and the impact of different well types in both isotropic and orthotropic media.

By showing the usability of PyEFVLib for this application, the study underscores the importance of Python as a versatile programming language with extensive libraries tailored for scientific computing. The successful simulations presented in this paper pave the way for more accurate and reliable reservoir management practices, ultimately contributing to more efficient and sustainable hydrocarbon extraction.

Methodology

In this topic, we discussed about the steps of grid generation, the modeling of the hydraulic diffusivity equation for a 2D single-phase fluid flow through an isotropic and orthotropic media with injection and production wells and its discretization using EbFVM.

The hydraulic diffusivity equation (HDE) for a single-phase fluid flow, with a non-compressible fluid and non-deformable porous medium is represented by,

$$\nabla \cdot (\mathbf{K}\nabla P) + q''' = \mu\phi c_t \frac{\partial P}{\partial t} \tag{1}$$

where, q''', μ, c_t, ϕ are the generation term which are the wells, viscosity, total compressibility, porosity respectively, and $\boldsymbol{K}$ is the permeability tensor in an anisotropic media, represented by (Ribeiro, 2010),

$$\boldsymbol{K} = \begin{bmatrix} K_{xx} & 0 \\ 0 & K_{yy} \end{bmatrix} \tag{2}$$

As described by Hurtado (2011), we need to integrate the Equation (1) in the control volume (CV),

$$\int_{t}^{t+\Delta t} \int_{\Omega_i} [\nabla(\boldsymbol{K}\nabla p) + q'''] \, d\Omega_i \, dt = \int_{t}^{t+\Delta t} \int_{\Omega_i} \mu\phi c_t \frac{\partial P}{\partial t} d\Omega_i \, dt \tag{3}$$

applying the Gauss's Divergence Theorem,

$$\int_{t}^{t+\Delta t} \int_{\Gamma_i} (\boldsymbol{K}\nabla p) \cdot \hat{n} \, d\Gamma_i \, dt + \int_{t}^{t+\Delta t} \int_{\Omega_i} q''' d\Omega_i \, dt = \int_{t}^{t+\Delta t} \int_{\Omega_i} \mu\phi c_t \frac{\partial P}{\partial t} d\Omega_i \, dt \tag{4}$$

the integrations can be approximated by,

$$\mu\phi c_t \frac{1}{\Delta t} p_p \, \Delta\Omega_i - \sum_{f \in \Gamma_i} \sum_{ip \in f} \boldsymbol{K} \overrightarrow{\Delta s_{ip}^{p}} \mathbf{B_{ip}} \overrightarrow{P^e} = q''' \Delta\Omega_i + \mu\phi c_t \, p_p^{old} \frac{1}{\Delta t} \Delta\Omega_i \tag{5}$$

the sum of the integration points in each face is the approximation of the integral. Besides that, the other summation is the sum of the faces in the control surface, which represents the sum of each integral approach in a CV.

The terms of the Equation (5) are indicated below:

- $\overrightarrow{P^e}$: represents the pressure vector associated on the nodes that share an element.
- p_p^{old}: is the pressure in the p node on the previous time step.
- p_p: is the pressure in the p node on current time step.
- $\mathbf{B_{ip}}$: represents the numerical operator that contain mathematical aspects capable of approaching numerically a scalar gradient as described by Hurtado (2011).
- $\overrightarrow{\Delta s_{ip}^{p}}$: is the area vector perpendicular to the area through which the flux passes, for each integration point at node p, with the vector magnitude representing the area value.
- $\Delta\Omega_i$: represents the volume of the control volume.

The calculation of these described variables $\left(\mathbf{B_{ip}}, \overrightarrow{\Delta s_{ip}^{p}}, \Delta\Omega_i\right)$ on Equation (5) is provided by PyEFVLib. The EFVLib library was developed as part of the first project sponsored by the Reservoir Simulation and Management Network (SIGER) from Petrobras at the SINMEC laboratory and one of the main objectives was to establish the foundation for applying numerical methods on unstructured meshes to the simulation of oil reservoirs (SINMEC, 2024).

As we have the discrete equation, it's possible to create a grid and identify the boundary conditions. The creation of an unstructured grid is an operation that involves a high complexity level, in which this task is developed through specialized software. They have sophisticated algorithms ensuring the grid generation with specifics requirements for a given numerical formulation (Hurtado, 2011). The scope of this paper is not to describe a grid generation algorithm, therefore it was used GMSH (Geuzaine et al., 2009), which is a software that develop grids ensuring the specification of the creation parameters.

Another factor considered in this paper is the well model. The Peaceman model (Peaceman, 1983) was selected, incorporating an approximation of the equivalent well radius, r_{eq},

$$r_{eq} \approx 0.208\sqrt{\Delta\Omega_i} \qquad (6)$$

Regarding an unstructured grid, we have approached the discrete length dimension used in Peaceman model (Peaceman, 1983), as the CV volume cubical root, in the associated well node. After that, it was developed a code in python capable of solving the problem described with the help of PyEFVLib.

Results and Discussion

The following section presents the results of our study, highlighting the key findings and their implications. Comprehensive data analysis was conducted to ensure the accuracy and reliability of the results. They have shown a solution to the HDE for a single-phase fluid flow, with a non-compressible fluid, non-deformable anisotropic porous media with 2 wells, a production and an injection well.

It was proposed a simple triangular grid to better understand the applicability of PyEFVLib. This grid is represented in Figure (1).

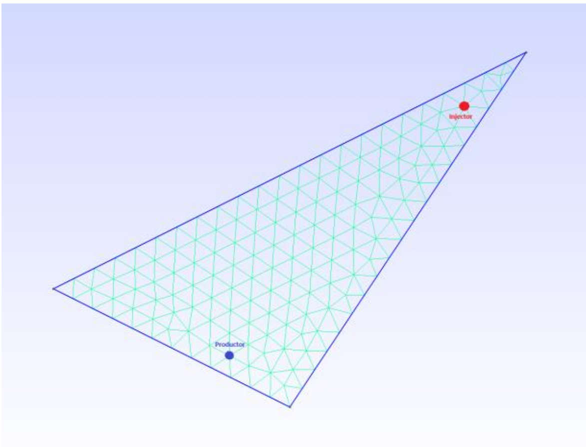

Figure 1 – Triangular grid

The mesh properties are detailed in Table (1).

Grid Properties	
Nodes	161
Elements	320
Boundaries Conditions	zero-flow

Table 1 – Grid Properties

The production well is located at node 128, while the injection well is at node 129. It should be emphasized that the arrangement of the nodes was created using the GMSH software (Geuzaine et al., 2009).

The well consideration varies with the type of it, for production it was designed the imposed pressure regarding the impossibility of a constant flow rate (Lee et al., 2003). For injection, the imposed flow rate was designed, aiming the easier capability of establish a constant flow rate in this type of well. The production-imposed pressure and injection-imposed flow rate are shown in Table (2) and their well radius (r_w).

Property	Productor	Injector
r_w (m)	0.01	0.01
Flow Rate (m³/s)	-	0.01
Pressure (Pa)	8.6e9	-

Table 2 – Well Properties

Another important consideration is the fluid and reservoir data, available in Table (3).

Property	Reservoir	Fluid
c_t (1/Pa)	-	2.04e9
μ (Pa s)	-	0.001
ϕ (-)	0.2	-
K_{xx} (m²)	1e-14	-
K_{yy} (m²)	0.5e-14	-
Initial Pressure (Pa)	1.2e10	-

Table 3 – Fluid and Reservoir Properties

For the numerical parameters, the time step considered is 0.1 seconds and the max number of iterations is 1000.

Consequently, the results were provided along the grid described in Figure (1). We analyzed three moments, the first time step ($t = 0.1$ s), an intermediate time step ($t = 79$ s) and close to the steady state ($t = 349$ s), as described in Figure (2), (3) and (4), respectively.

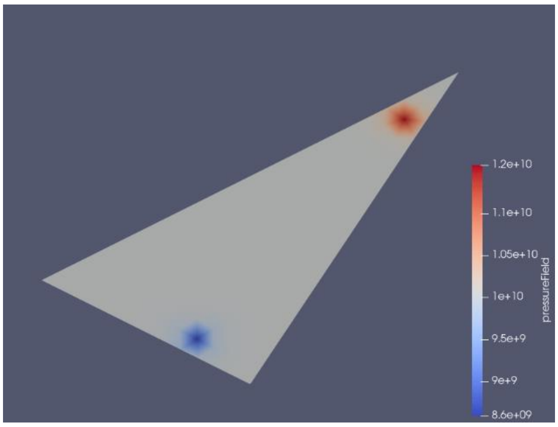

Figure 2 – The pressure field along the reservoir after 0.1 seconds of simulations

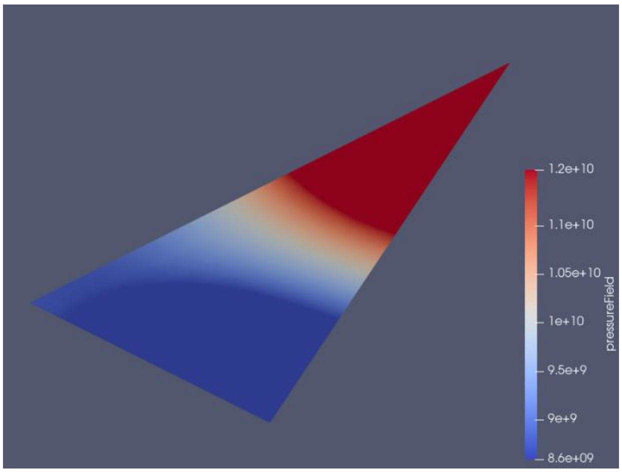

Figure 3 – The pressure field along the reservoir after 79 seconds of simulations

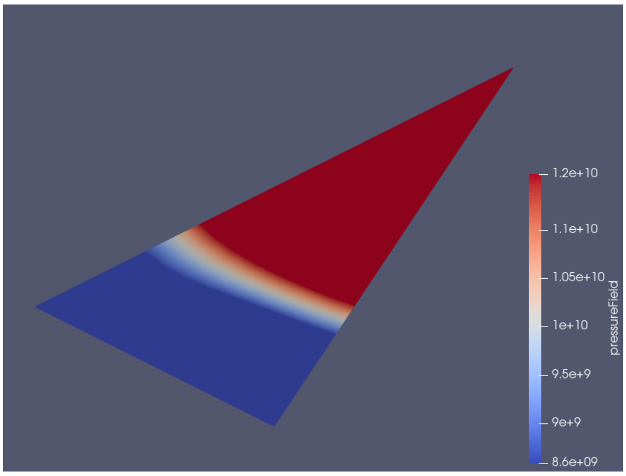

Figure 4 – The pressure field along the reservoir after 349 seconds of simulations

As time progresses in the simulation, the pressure provided by the injection well begins to increase the reservoir pressure. Conversely, the energy drained by the production well decreases the influence of the reservoir pressure. Additionally, a state close to steady state is formed, indicating the zone of influence of the wells within the reservoir boundaries.

The results showed that the finite volume element-based method applied through PyEFVLib is consistent with the expected outcomes of a simulation for the specified problem. This validates the use of the library in the context of applying the EbFVM method.

Conclusions

The present work is a demonstration of the usage of a finite volume method, specifically the element-based finite volume method, through a Python library, PyEFVLib. This method is employed to tackle complex simulations within the context of reservoir engineering.

Throughout the paper, the governing Hydrodynamic Equation (HDE) is defined, which describes the physical phenomena proposed by the authors. This includes the representation of a single-phase flow in a 2D anisotropic porous medium, taking into account various considerations such as boundary conditions, initial conditions, and specific parameters relevant to the reservoir's properties.

The results presented in this study validate the use of the Python library, PyEFVLib, by demonstrating the accurate simulation of pressure evolution over time within a triangular reservoir. The study not only showcases the capability of PyEFVLib in handling such simulations but also highlights its efficiency and reliability in modeling the dynamics of reservoir behavior under specified conditions.

In conclusion, the findings confirm that PyEFVLib is a robust tool for applying the element-based finite volume method in the field of reservoir simulation, providing a solid foundation for further research and practical applications.

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