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TÍTULO DO TRABALHO:

Numerical simulation of single-phase flow in a three-dimensional heterogeneous reservoir

AUTORES:

Tiffany Cristine Franzoi, Guilherme Willemann Alberton, Lindaura Maria Steffens

INSTITUIÇÃO:

Universidade do Estado de Santa Catarina

NUMERICAL SIMULATION OF SINGLE-PHASE FLOW IN A THREE-DIMENSIONAL HETEROGENEOUS RESERVOIR

Resumo

A dinâmica do fluxo em meios porosos constitui uma área significativa de estudo na indústria de petróleo e gás, servindo como uma ferramenta essencial para o setor. Os reservatórios são sistemas altamente complexos, e a capacidade de modelar com precisão seu comportamento é crucial para a minimização de riscos e otimização da produção. Portanto, a análise de aproximações numéricas torna-se um componente essencial para prever o comportamento dos reservatórios.

Usando o Método das Diferenças Finitas (MDF) com uma formulação implícita (amplamente utilizada por sua estabilidade incondicional) para modelar a equação diferencial parcial conhecida como Equação da Difusividade Hidráulica (EDH), derivada da equação de conservação de massa e da Lei de Darcy, é possível obter o campo de pressão, uma variável fundamental para tais estudos. Além disso, são abordadas condições de contorno e inicial para capturar o comportamento do sistema poço-reservatório para análise. Isso permite a avaliação de dados relacionados à viabilidade do projeto, regimes de fluxo, avaliações futuras de reservas e muito mais.

O principal objetivo deste trabalho é apresentar o desenvolvimento de um simulador *in-house* de reservatório, descrevendo o fenômeno físico modelado, o modelo matemático correspondente, bem como o método numérico para resolução e as respectivas implementações. Em alguns casos, suposições simplificadoras podem ser empregadas para obter essas respostas de maneira mais eficiente, mantendo a fidelidade ao comportamento do mundo real. Portanto, no presente estudo, para representar a realidade de forma mais fiel, é considerado um reservatório com porosidade homogênea e permeabilidade heterogênea com fluxo monofásico em um espaço tridimensional, refletindo as complexidades geológicas encontradas em problemas realistas (MACHADO, 2024). Para abordar isso, o MDF implícito foi aplicado para obter a solução e análises numéricas foram conduzidas usando rotinas baseadas em Python, descrevendo o problema de várias maneiras distintas.

Dado que as soluções numéricas são aproximações, é essencial que a análise dos resultados seja guiada por um julgamento de engenharia sólido para conduzir estudos de reservatórios de maneira eficaz e interpretar os resultados com precisão (AZIZ; SETTARI, 1979). Assim, o simulador desenvolvido permite determinar o comportamento do reservatório sob diferentes circunstâncias, incluindo a implementação de possíveis poços e falhas, gerando diferentes resultados e gráficos que permitem diferentes análises e interpretações.

Palavras-chave: reservatório tridimensional, simulações numéricas, método das diferenças finitas.

Abstract

The dynamics of flow in porous media constitute a significant area of study in the oil and gas industry, serving as an essential tool for the sector. Reservoirs are highly complex systems, and the ability to accurately model their behavior is crucial for risk minimization and production optimization. Therefore, the analysis of numerical approximations becomes an essential component for predicting reservoir behavior.

Using the Finite Difference Method (FDM) with an implicit formulation (widely used for its unconditional stability) to model the partial differential equation known as the Hydraulic Diffusivity Equation (HDE), derived from the mass conservation equation and Darcy's Law, it is possible to obtain the pressure field, a fundamental variable for such studies. Additionally, boundary and initial conditions are addressed to capture the behavior of the well-reservoir system for analysis. This allows for the assessment of data related to project feasibility, flow regimes, future reserve evaluations, and more.

The main objective of this work is to present the development of an in-house reservoir simulator, describing the modeled physical phenomenon, the corresponding mathematical model, as well as the numerical method for resolution and the respective implementations. In some cases, simplifying assumptions can be employed to obtain these responses more efficiently while maintaining fidelity to real-world behavior. Therefore, in the present study, to represent reality more faithfully, a reservoir with homogeneous porosity and heterogeneous permeability with single-phase flow in a three-dimensional space is considered, reflecting the geological complexities found in realistic problems (MACHADO, 2024). To address this, the implicit FDM was applied to obtain the solution and numerical analyses were conducted using Python-based routines, describing the problem in various distinct ways.

Given that numerical solutions are approximations, it is essential that the analysis of the results be guided by solid engineering judgment to effectively conduct reservoir studies and interpret the results accurately (AZIZ; SETTARI, 1979). Thus, the simulator developed allows determining the behavior of the reservoir under different circumstances, including the implementation of possible wells and failures, generating different results and graphs that allow different analyzes and interpretations.

Keywords: three-dimensional reservoir, numerical simulations, finite difference method.

Introduction

The dynamics of flow in porous media constitute a significant area of study within petroleum reservoir engineering. Using the partial differential equation, shown in Equation 1, known as the Hydraulic Diffusivity Equation (HDE), derived from the mass conservation equation and Darcy's Law, it is possible to obtain pressure, a fundamental variable for such studies. Additionally, boundary and initial conditions are addressed to capture the behavior of the well-reservoir system for analysis. This allows for the assessment of data related to project feasibility, flow regimes, future reserve evaluations, and more.

In some cases, simplifying assumptions can be employed to obtain these responses in a more streamlined manner while maintaining fidelity to real-world behavior. In the present study, homogeneous and heterogeneous permeability reservoirs were considered, both three-dimensional with single-phase flow. To address this, numerical analyses were conducted using Python-based routines. The main objective was to interpret the resulting graphs to determine the reservoir's behavior, implementing the possibility of wells within the reservoir.

Moreover, the study enfatize the importance of using computational tools in reservoir simulation, showcasing how Python's libraries can effectively handle complex mathematical computations and data visualizations. The use of SciPy (VIRTANEN *et al.*, 2020) and NumPy (HARRIS *et al.*, 2020) facilitated the solving of intricate equations, while PyVista (SULLIVAN; KASZYNSKI; WILSON, 2023) enabled the construction of a graphical representations. This integration of tools highlights the potential for Python to be a robust platform for reservoir engineering simulations, offering a balance between computational efficiency and accuracy.

Methodology

For the numerical models, were consider a reservoir 3D of size equal size in all directions ($Lx, Ly\ Lz$), with an initial pressure p_0 at the initial time and a production well with bottom pressure p_w. Additionally, were assumed constant permeability (k) in the case of a homogeneous reservoir and varying permeability between blocks for the case of a heterogeneous reservoir. Other reservoir properties such as porosity (ϕ), viscosity (μ) and (c_t) are considered constant. Furthermore, all simulations consider a no-flow boundary condition.

The formulation used in this work was implicit by the Finite Difference Method (FDM), as shown in Equation 1, as it does not require a stability criterion. This type of formulation is centered in space and backward in time, necessitating the solution of a linear system through the generated matrix. The module used for solving the linear system was *linalg* from NumPy, which contains standard linear algebra operations. The implicit FDM approach offers robustness and stability, making it suitable for the complex simulations required in reservoir modeling.

$$\phi c_t \frac{\partial p}{\partial t}\bigg|_{i,\,j,k}^{t+\Delta t} = \frac{\partial}{\partial x}\left(\frac{k}{\mu}\frac{\partial p}{\partial x}\right)\bigg|_{i,\,j,k}^{t+\Delta t} + \frac{\partial}{\partial y}\left(\frac{k}{\mu}\frac{\partial p}{\partial y}\right)\bigg|_{i,\,j,k}^{t+\Delta t} + \frac{\partial}{\partial z}\left(\frac{k}{\mu}\frac{\partial p}{\partial z}\right)\bigg|_{i,\,j,k}^{t+\Delta t} + \frac{2\pi k_w}{\phi \mu c_t}\frac{\left(p_{wf} - p_{req}\right)}{\Delta x \Delta y ln\left(\frac{r_{eq}}{r_w}\right)}$$

Equation 1 – Hydraulic Diffusivity Equation.

Heterogeneity in the cases is addressed through the concept of transmissibility, which considers the exchange of information between blocks and is a function of permeability, viscosity, and geometry (ROSA; CARVALHO; XAVIER, 2006). This approach ensures that the variability in permeability is accurately represented, allowing for a more realistic simulation of the reservoir behavior. By calculating transmissibility using the harmonic mean to determine the equivalent permeability between blocks, the model accounts for the differences in flow capacity between adjacent blocks, which is critical in heterogeneous reservoirs.

Results and Discussion

Considering the property inputs and mesh definitions provided in Table 1, it was possible to obtain the pressure behavior for the cases proposed. These detailed inputs allowed for precise modeling of the reservoir conditions, enabling us to simulate various scenarios and analyze the resulting pressure distributions comprehensively.

Reservoir Properties	Constants	Mesh Properties	Constants
p_0 [kgf/cm²]	1300	$Matrix$	*10x10x10*
p_w [kgf/cm²]	*900*	Δt [s]	*0.0005*
μ [cP]	*2.1*	t_{max} [s]	*40*
ϕ [−]	*0.2*		
c_t [(kgf/cm²)⁻¹]	*9e-7*		
L_x, L_y, L_z [m]	*800*		

Table 1: Reservoir and mesh properties used.

For the first case study, the permeability map shown in Figure 1 is considered. Thus, the simulation grid blocks contain the same value of 200 mD for the property, resulting in the case of reservoir with homogeneous permeability. The pressure behavior for this case is presented in Figure 2.

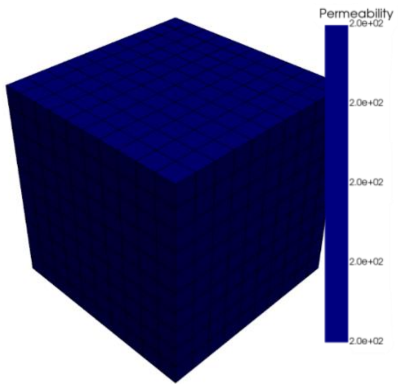

Figure 1 – Homogeneous permeability map.

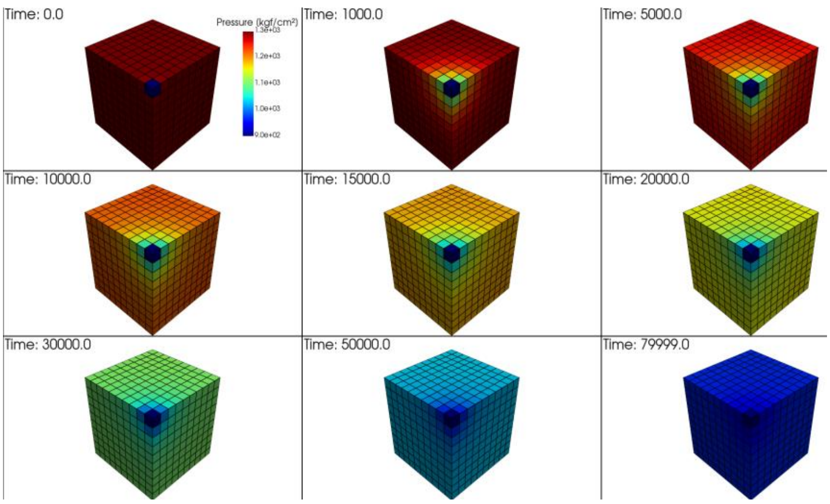

Figure 2 – Pressure field for homogeneous permeability map.

By analyzing the pressure field graph presented in Figure 2, an isotropic behavior of the pressure variable is observed. This behavior is expected, as the permeabilities are uniform, resulting in no preferential spatial direction for fluid flow. Consequently, the pressure drop is proportional to the distance from the block to the well.

In the second case, a heterogeneous permeability map was generated using Excel and manually adjusted to create two distinct zones: one with higher permeability and another with lower permeability values. This map is shown in Figure 3, and Figure 4 present the pressure field (blue means low pressures and red higher pressures).

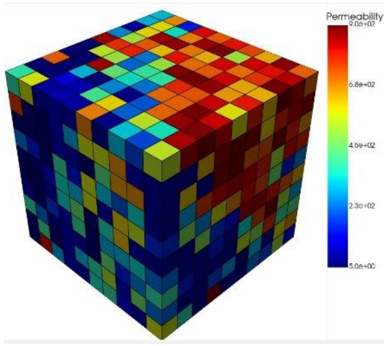

Figure 3: Heterogeneous permeability map.

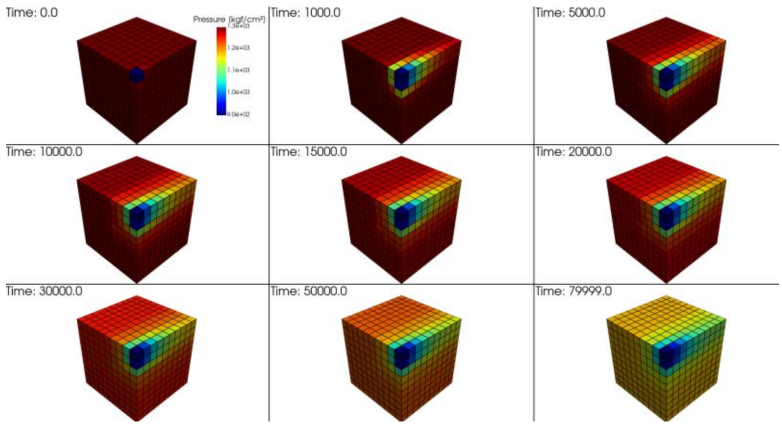

Figure 4: Pressure field for heterogeneous permeability map.

As expected, the behavior of the pressure field is now distinct from the previous case. This is due to the fact that this permeability map has a zone with high permeability, which facilitates fluid flow within the reservoir, resulting in a more depressurized zone compared to the zone with lower permeability.

The final case considers the previously presented heterogeneous permeability map with the addition of a fault, which characterizes a zone where the permeability is zero, meaning there is no flow through it. This map is shown in Figure 5 and the pressure field is presented in Figure 6.

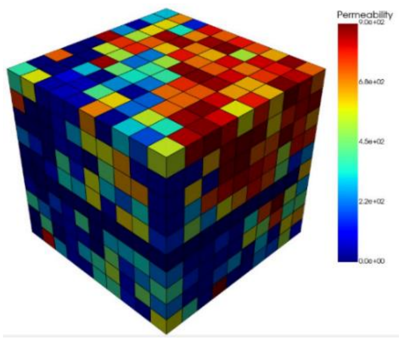

Figure 5: Heterogeneous permeability map with fault.

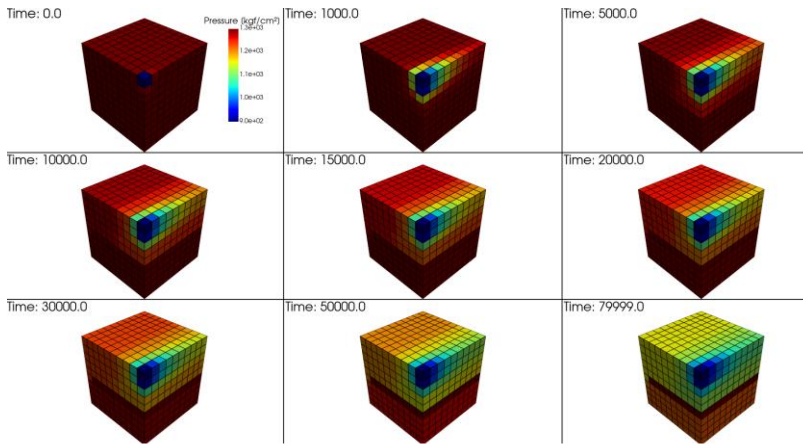

Figure 6: Pressure field for heterogeneous permeability map with fault (case 1).

The pressure behavior remains similar to the previously presented case, but now the fault provides a sealing barrier; however, there is still connectivity with the lower part. Consequently, the fluid cannot pass through it, causing the zone affected by the fault to take longer to depressurize, as the fluid now needs to travel a greater distance to reach the well.

To demonstrate a new scenario, using the same permeability map from Figure 5 and the same fault model, the position of the well was moved closer to where the fault opens, in a region of high permeability. Figure 7 shows the new pressure field generated.

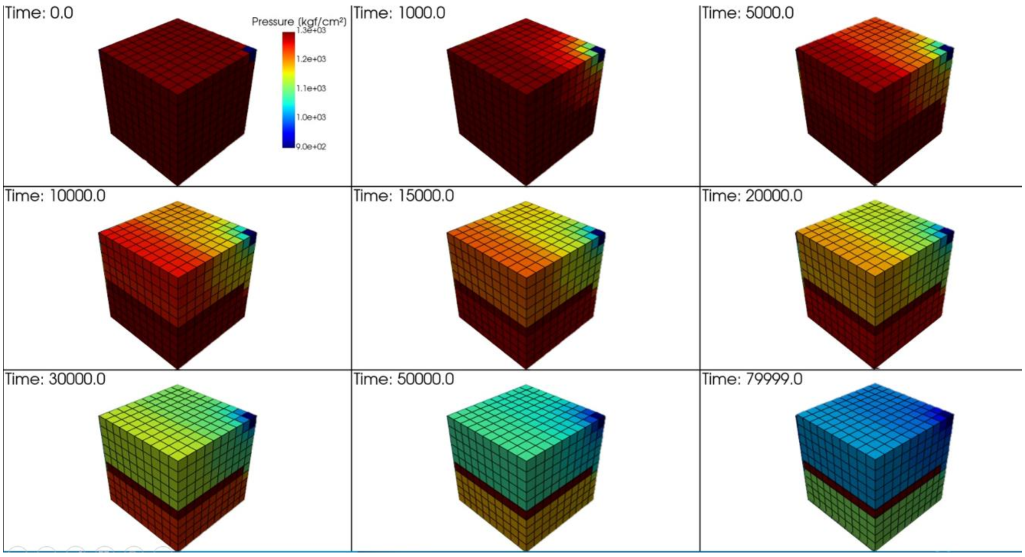

Figure 7 - *Pressure field for heterogeneous permeability map with fault (case 2).*

From the new pressure field, it is possible to observe that as time passes, the reservoir drains more of the fluid contained within it. This, in practical terms, defines the best location for a well to achieve higher production.

Furthermore, to demonstrate the computational demand, the simulation times for all simulated cases were compared, as shown in Table 2.

Case	Time [min]
Homogeneous	*43’56’’*
Heterogeneous without fault	*44’47’’*
Heterogeneous with fault (case 1)	*38’94’’*
Heterogeneous with fault (case 2)	*40’68’’*

Table 2: Time comparison.

From the table, it is evident that the computational time is shorter in cases where there is a failure in the reservoir. This is due to the numerical process: when there is a failure, the permeability is zero, meaning the block does not need to solve the equation or go through the numerical process (it will be zero at all time steps).

Conclusions

This academic study demonstrated the coherent application of numerical approaches for monophasic (applying FDM) fluids, aligning with the expected behavior of a reservoir and providing valuable insights. The analysis included graphical representations of production, aiming to predict optimal well locations for maximizing oil production.

Furthermore, the study delved into the presence of faults within the reservoir, allowing for an exploration of their impacts on reservoir behavior. The comprehensive examination showcased the robustness of the numerical method employed, offering a nuanced understanding of reservoir dynamics.

In summary, this research not only validated the numerical methodology for three-dimensional reservoir scenarios but also contributed valuable insights into predicting optimal well placements for enhanced oil production. The exploration of fault effects added an additional layer of complexity to the analysis, highlighting the study's significance in advancing our understanding of reservoir behavior.

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